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SCIENCE and PHILOSOPHY

THE CONCEPT OF TIME

By REV. JOSEPH P. KELLY, S.J.

"... we find that Time is the most mysterious problem that our mind can try to solve. This thing which is and no longer exists, which perishes and remains; this immobile flight, this changing continuity, the long wake of a terrible course, what is it, in truth?" Nys. "La notion de temps." p. 9.

What is Time? We tell time, the clock tells time. There are good times and bad times. We save time, we lose time. In the C. G. S. system time is a fundamental unit. There are innumerable expressions of this sort familiar to all. They are acceptable in daily life. Through them we communicate our thoughts to one another. Everyone has some knowledge of time, somewhat vague perhaps, but at least that knowledge which one acquires from the use of terms without much reflection on their meaning. The common use of the word, Time, would seem to indicate a fundamental significance underlying these notions. (1). If one were to try to explain its meaning, he might feel as St. Augustine did when he said: "If no one asks me, I know; if I wish to explain to one that asketh, I know not." (2). Not all the world's thinkers held the view of Augustine, if one were to judge from the literature on this subject. Among the Greeks, we find Aristotle discussing the problems of time. (3). In the Middle Ages, St. Thomas Aquinas, Suarez and others wrote treatises on this mysterious question. (4). Modern philosophers and scientists, e.g., Descartes, Kant and Newton have added their contributions. And today, Time is still a widely discussed question both in science and philosophy. (5). Let us note here that in this article, we are dealing with *Time in itself* and not with the *measure* of time. These are two distinct problems. Mathematics has a large share in modern Physics. Because of this, measurement has assumed tremendous importance in Science, so much so that many scientists would exclude from their field of investigation whatever is not measurable. There is a strong tendency to look upon the measurement of an object as the object

- (1) As a small problem in the meaning of terms: What would be the precise significance of the term, Time, as used in these expressions?
- (2) St. Augustine. "Confessions". Bk. XI, c. 14.
- (3) Aristotle. "Naturalis Auscultationis."
- (4) St. Thos. "Opus. de Tempore." Suarez. "Disp. Meta." D. 50. "De Duratione." Sect. VIII.
- (5) A bibliography on "Time" will be added later.

itself. (6). The accurate measurement of physical phenomena belongs to the world of the scientist. It contributes much to progress in the Natural Sciences. But no measurement process can reveal the nature of objects. However much it assists in the description of the physical world, it cannot reveal the intrinsic nature of it.

Time, by which we order our daily life and actions, is based on the motion of the earth. One complete rotation of the earth on its axis constitutes the fundamental unit of time; it is called, a day. For human convenience, it is divided into hours, minutes and seconds. This division is an arbitrary one, suited to practical life. Since the rotation of the earth is unceasing, the determination of one complete rotation must be reckoned from some chosen place, e. g., Greenwich or Washington. Again, we have the sidereal day and the solar day, according to the method of determining the single rotation. These, however, pertain to the measurement of time, rather than to time itself. What concerns our problem of time is the rotatory motion of the earth and not the various methods of measuring it. We cannot determine time by mere observation, with any degree of accuracy. Hence, various chronometric instruments, clocks, watches, etc., have been devised to indicate time. These instruments have been so constructed that their movements are more or less in harmony with that of the earth. We interpret the movements of our clocks in terms of time. We "tell" time by them.

TIME AND MOTION

Time and motion are intimately connected; in fact, they are in a way identified. Time is motion under a certain aspect. Time is motion measured. Motion is the transition from one state of being or place to another state or place. This passage from one state to another is called change or in a broad sense, movement. The change from place to place is local motion. It will be more in keeping with our purpose to limit the discussion to local motion, first, because it is more familiar to us and secondly, because our ordinary Time is based on local motion. Generally speaking, these transitions do not happen all at once. They have duration, they last for an appreciable interval. Motion is a continuous achievement from the qualities of motion. From these we derive the notions of extension and of part succeeding part. Just as in the rotation of the earth, we can consider the first quarter of rotation, then the second quarter, etc., as taking place in succession. We realize that in the concrete, the motion of the earth is one, continuous movement. Mentally, we can designate the succession of parts. Hence, in the actual unity of motion, we can mentally recognize a multitude of parts, one following another. Motion is objectively one, but virtually multiple. When we break up

(6) Newton, in his "Principia" warned the scientist of this dangerous trend. c. f. "Principia." p. 11.

a movement so as to distinguish what goes before and what comes after, we have the notion of Time. "Time is the numbering of motion according to a relationship of before and after." (7). Time is the measure of the duration of motion. Time is motion, considered not as a transition from one place to another, but as the enumeration of the succession of parts. The rotation of the earth on its axis is continuous; it is motion. When we look at this motion under the aspect of one rotation after another, we number the rotations, as it were, and we have the fundamental unit of Time, the day. The "hours" are nothing more than our arbitrary division of the unit rotation into twenty-four parts, in succession. Convenience apart, the motion might equally well be divided into one hundred or one thousand parts as well as twenty-four. The same may be said, in proportion, of the division of the hour into sixty minutes and the minute into sixty seconds.

IS TIME REAL

Time is the measure, (the numbering of parts) of motion, according to a relationship of before and after. In the concrete reality, time and motion are identified. We can place only a logical distinction between them. We say a logical distinction because according to the definition, the formal concept of motion is the transition from place to place. (This is for local motion). But since it has duration and continuity, we are able to look at it in another way, viz: as having a succession of parts, the "before and after" relationship. When we thus divide movement into its potential parts, the process belongs to the mental order. The temporal aspect of motion is a creation of the mind. It does not actually separate part from part in the physical order. The motion remains the same. The rotation of the earth goes on as one continuous motion, no matter how the human intellect may divide it. The enumeration of parts is a mental process, and this constitutes Time in its formal concept. It is not a mere mental category, a *purely subjective form* which enables the intellect to perceive physical phenomena in a temporal relationship. This is what Kant and his followers would have us believe. Nor, on the other hand, must we go to the opposite extreme with Newton and the majority of Classical physicists who have attributed to Time, a reality of its own, independent of all other things. From our analysis, it follows that Time is partly real and partly ideal. Because of its identity with motion, in the physical order, it has the same reality as motion. It is *materially* identical with motion. But the formal concept of Time is not the same as the formal concept of motion, as we have seen. Both have their proper definitions differing one from the other. The "numbering" of motion is a mental process. Hence Time, in its formal aspect, is ideal, a being of the mind. Time is an abstract from motion. Time is a being of the mind with a foundation in an objective reality.

(7) c. f. Suarez. "Disp. Meta." D. 50; Sect. VIII.

One might question the validity of our notion of Time, on the score that in the physical order there is a considerable variety of motions: rapid, slow, translatory, vibratory, periodic, etc. Will these determined forms of motion affect the universal concept of Time? We do not believe so. The idea of time does not deal with the special types of motion as such. Time has a relationship to motion as motion, not to motion as a particular form of movement. For example, let us suppose that the earth were to rotate on its axis twice as fast as it does at present. Would the concept of Time remain the same under these new conditions? We would have a day of twelve hours, preserving the present notion of an hour, or a twenty-four hour day, if we would shorten the hour to one-half of its present length. But on the other hand, we would still have a fundamental unit of Time, one complete rotation of the earth on its axis. We could still preserve our definition of Time. Rapidity or slowness or any other determined form of motion cannot affect the essential concept of Time. They may affect only our mode of enumeration of parts. Time may be perceived in any form of motion. In the analysis of motion the mind prescind from all the various types of motions and fixes its attention on the one common element in all of them. It sees a successive continuity, a relationship of before and after. This element gives us the justification for the formal and universal concept of Time and its logical distinction from motion as such.

(to be continued)

Science for December 15th, 1944, notes the post-war building plan for St. Louis University School of Medicine includes two new buildings, one for the physics and geophysics departments, and the other for the Chemistry and Biology departments.

BIOLOGY

PREFRONTAL LOBOTOMY IN THE TREATMENT OF PSYCHOSIS

By REV. PHILIP H. O'NEILL, S.J.

It has long been known that the removal or stimulation of the prefrontal areas of the human brain produces no observable motor or sensory changes. (1). Frequently, however, this treatment affects the entire personality. Cases of brain tumor, accidental and self-inflicted injuries furnished enough evidence to suggest that the prefrontal area is a control center which contributes foresightedness and critical judgment to human behavior. The hyperactivity of this region of the brain, according to a reasonable hypothesis, might be the cause of various types of depression and obsession. At the Second International Neurological Congress in London, August, 1935, the function of the prefrontal areas was one of the principal topics of discussion. Present at the congress was Egas Moniz of Lisbon, who had already formulated his theory that a leucotomy in the anterior region of the brain would release the patient from pathological inhibition in cases of depression psychosis. He performed the first operation of this type on November 12, 1935—a notable event in the history of surgery and psychiatry, for it may be considered the first "valid operation for mental disorder." (5). In this country the first prefrontal lobotomy was performed by Freeman and Watts September 14, 1936 in Washington, D. C.; later, in June, 1937, J. G. Lysterly of Jacksonville began to perform the same type of operation.

Since that time many case histories have been reported in the literature. The results of the operation have been both good and bad. No one has claimed that prefrontal lobotomy can effect a cure by which the patient is returned to his normal condition, but the operation is advocated and performed on the grounds that certain types of psychotics are much better off after they have been freed from the excessive inhibition of the prefrontal area. The general improvement in the patient's condition, it is claimed, more than offsets the undesirable changes in personality which usually follow the operation. In this paper we will discuss the subject very briefly under the following headings: I. The general theory underlying the operation. II. The method of performing the operation. III. The results of the operation.

(1) This paper was read at the Summer Meeting of the American Association of Jesuit Scientists, Maryland and New York Provinces.

I. The general theory underlying the operation. Prefrontal lobotomy consists in the surgical division of the white matter of both frontal lobes (6); more specifically, the region in question includes areas 9, 10, 11, 12, 45, 46, 47 on the lateral aspect of the hemisphere and areas 24, 32 and 33 on the mesial aspect. Roughly this includes the entire frontal lobe, anterior to the premotor area. It is important, therefore, to review the present state of our knowledge regarding the neurological connections of this part of the brain. Association fibers in great abundance, both afferent and efferent, communicate with the rest of the cerebral hemispheres; arcuate fibers just beneath the cortex connect neighboring convolutions while the funiculus cinguli, the uncinate fasciculus and many others extend to the more distant areas. Commissural fibers connect the prefrontal areas in the two hemispheres via the corpus callosum.

None of these fasciculi have been traced out in exact detail; still less is known about the projection fibers. Walker (15) has shown experimentally that in the macaque monkey "the small celled part of the medial dorsal nucleus sends fibers to the cortex around the inferior precentral and inferior prefrontal sulci." Having found the same connection in the chimpanzee, he believes that this relationship very probably exists in man. (16, pg. 192) Krieg (8, pg. 34) traces the path through the anterior thalamic radiation, the anterior thalamic radiation, the anterior limb of the internal capsule, to the prefrontal area, chiefly 9 and 10. He admits that these details are not definitely established. Krieg (pg. 340) also describes the connections, both afferent and efferent, between the prefrontal areas and the red nucleus; here again there is much uncertainty.

Such then, according to our present knowledge, are the neural pathways which are interrupted by prefrontal lobotomy. Moniz predicted that this severance of the old fixed pathways would force the brain to develop new circuits or at least remove the neural basis for the stereotyped obsessive thinking which is characteristic of certain psychosis. According to him, (11) the thought processes of a normal mind are able to shift freely and easily from one neural circuit to another; a psychosis occurs when one circuit becomes fixed and disappears when the circuit is broken by a leucotomy.

Freeman and Watts (5, pg. 26) attribute the principal effect of lobotomy to the cutting of the thalamic-prefrontal connections. The thalamus is known to be associated very closely with the emotions in many psychoses certain ideas become excessively charged with emotion and this leads to abnormalities in behavior. Moreover, the prefrontal areas according to Freeman and Watts (6) integrate the idea with relation to the self and the future, so that the consequences of the action are made evident. The ego perceives what "ought to be done," sees the outcome of this or that type of behavior, acquires foresightedness and a sense of responsibility. When such ideas become invested with an excess of emotion there follows a psychosis of the

compulsive, obsessive or paranoiac variety. Prefrontal lobotomy, as Freeman and Watts explain it, simply pulls the switch and breaks the circuit between the thalamus and the cortex, releasing the patient from harassing ideas about the future, or if the ideas persist after operation, they are without emotional charge. And of course the loss of this cortical control over behavior, is a serious disadvantage to the patient after the operation.

In evaluating these theories of Moniz, Freeman and Watts two important points should be kept in mind. First, although the speculative element bulks very large, neither theory lacks plausibility, especially when the actual results of the operation are considered. In a number of cases, the two theories seem to furnish a very exact description of the post-operational changes in personality. Secondly, the operation is highly empirical, partly because the neuroanatomy of the prefrontal area is so poorly understood. The surgeon permanently disconnects an important organ, the function of which has not been definitely determined, although it is known to be connected with man's highest and specifically human endowments. This constitutes a serious objection and yet it is not insuperable. One can hardly object to the procedure of Peterson and Buchstein who resort to leucotomy only in cases of chronic, long institutionalized, violently aggressive patients, after all other less drastic methods have been tried. At least in these circumstances, the possibility of success seems to justify the danger involved. To sum up, the factual and theoretical foundations of the operation are very shaky and the strongest arguments toward its justification must be drawn from the actual results. We will consider these in a later section.

II. The methods of performing the operation. We have only one purpose in discussing surgical methods, which after all are not a matter for critical appraisal by the layman. But anyone who reads the literature on prefrontal lobotomy can see that to a great extent it is a blind operation. The surgeon does not know exactly what he is cutting and this is an important fact in any study of the pros and cons of the operation.

The objective is to cut the white matter of the frontal lobe in the plane of the coronal suture. A burr hole is opened 3 cm. posterior to the lateral rim of the orbit and from 5 to 5.5 cm. above the zygomatic process. "A burr hole placed at this point passes through the coronal suture" (4), according to Freeman and Watts. Because of differences in individual skulls, however, this statement can only be true in general, with many exceptions. For better orientation a small incision is made in the midline about 12 to 13 cm. from the glabella in order to locate the intersection of the coronal and interparietal sutures. Important landmarks are the median longitudinal fissure and the sphenoidal ridge, which are located by probing with suitable instruments. Having thus located the guide-posts, Freeman and Watts strive

to cut about 80 to 90% of the fibers just anterior to the tip of the ventricle. This method keeps them out of the dangerous motor and premotor areas, but beyond this is seems a rather dubious guide. Lysterly (10) uses a two-bladed brain speculum to afford some visibility; Strecker, Palmer and Grant insert a hollow needle in order to find the tip of the ventricle with greater security. (13).

Each surgeon, of course, has his own methods, but all seem to be groping and cutting somewhat in the dark. This is indicated by the fact that sometimes the operation has to be repeated (3) when no results are apparent after the first leucotomy; the procedure in such cases is simply to insert the leucotome again and cut some more white matter. Obviously there is no question of cutting individual tracts or fasciculi. These facts again seem to indicate the empirical nature of the operation. The effects are not predictable on the basis of neuroanatomy and neurophysiology, but taking the statistical viewpoint, the surgeon can foretell the usual result, or the result in a certain percentage of the cases. The individual is not quite as important in this procedure as the patient would like to be considered when he goes to his physician. In justice to the surgeon, however, the literature indicates that the patient or his guardians are told before the operation exactly what the possibilities are. Incidentally, the operation has an extremely low mortality record, and although brain surgery suggests fearful connotations to most people, prefrontal lobotomy causes very mild physical disturbances. When Strecker, Palmer and Grant (13) refer to it as a "definitely brutal method" they are considering not the immediate physical results of the operation but the long range effects on the personality. The personality change is the crucial factor in any discussion of this type of psychosurgery.

III. The results of prefrontal lobotomy. No single case history can be offered as truly typical, but the following will serve as an example. H. D., a white man, aged 53 years, engineer and teacher. As a small child he was nervous and sleepless, lying awake imagining such horrors as awakening in one's grave, the terror's of Hell, etc. He always had a sensation in the right side of his head and kept twisting his face to relieve it. Taught school but failed because of nervous mannerisms and impatience. He succeeded for a time as a tool designer but the sensation grew until he was almost crazed and he had to discontinue this work. For 18 months prior to the operation he was supported by his wife and could do nothing but childish tasks around the house. He would talk with much snorting, little moans and shrieks, pointing of the fingers and many mispronunciations. He worried constantly, was very restless and given to temper tantrums.

Prefrontal lobotomy was carried out on May 12, 1937. On May 25 he was able to start for home driving his own car in Washington traffic. (!) Eight months later all his friends agreed that he had improved immensely. Worry and facial grimaces had disappeared; many of his skills and abilities, such as playing the violin had been complete-

ly recovered; he was doing useful work but could not get a regular job because of the depression. (3).

Similar successes have been reported in cases of schizophrenia, ruminative obsessive psychoses and involutional melancholia. When the chances of success are being weighed it is better to ignore the diagnostic labels of the disease and look for such symptoms as fear, anxiety, acute mental suffering and aggressive violence; in such cases, according to Grant (13) leucotomy is very likely to be helpful. Lyerly (9) claims success in 14 out of 14 cases of involutional melancholia; the majority of these patients, however, had been suffering from the disease for less than a year and one may wonder whether other methods had been tried, such as the injection of sex hormones (16). Heilbrun and Hletko could see only slight improvement in 2 out of 10 victims of schizophrenia who had been subjected to the operation (7). Petersen and Buchstein, however (12), performed the operation on 25 schizophrenics with great success. The patients had been living in institutions for many years and were universally regarded as hopeless. In 12 cases there was marked improvement and 6 of the patients were able to go home. Strecker, Palmer and Grant (13) were able to help 3 out of 5 greatly deteriorated schizophrenics, 2 of whom were able to return to their homes. Out of 16 cases of agitated depression, 12 improved to such an extent that they were able to return home. All of these patients had suffered for many years and had failed to respond to any other treatment. The literature on this subject is growing rapidly as more and more surgeons take up the operation but the foregoing reports are fairly representative and will suffice to show the chances of "success or failure." Success in this context merely means an amelioration and it is better to examine the results in detail than to accept a catagorical label.

Within a few days after the operation the patient is perfectly orientated in time and place; hallucinations, worry and grimaces have disappeared. The countenance is dull and unemotional with a mask-like expression; the voice too is emotionally flat. Emotional flexibility gradually returns and with it come euphoria, lack of self-consciousness and loss of inhibition. The patient is apt to talk offensively without any thought of consequences. Trifling events may cause unrestrained mirth. As a rule mental tests show that there is little if any intellectual deficit, or there may even be an improvement. The individual tends to live strictly in the present. A telephone operator, for instance, could handle calls perfectly but often forgot to keep a record of them afterwards; when being tested on a maze, one patient fell into a rut and made the same error on every trial. There is often an increase in appetite and a gain in weight. If obsessive ideas persist, they no longer cause trouble because they have lost their emotional coloring. The patient cannot pursue a definite course of action to reach a goal; he may spend two or three hours in the bath when his original intention was to prepare for an engagement.

Exceptions can be found for every one of the foregoing statements. One patient was riding horseback in the company of a friend when the latter suffered a serious accident; she was able to administer first aid effectively and to summon help. Another patient is now earning his living by driving a truck. One wife writes to thank the surgeon for working a miracle; another declares that "he can think of no surer way to ruin a whole family than prefrontal lobotomy. After analyzing many case histories, Cobb concludes that "it still remains difficult to describe typical frontal lobe symptoms." (2).

In the more recent literature, one very important new factor is emerging; psychosurgery alone is not sufficient to bring about a cure. "Postoperational re-education is imperative" (13) and a great deal depends upon the type of treatment which the patient receives during the years which follow the operation. The immediate postoperational state, according to the vivid statement of Freeman and Watts (6), is a "surgically induced childhood." New adjustments can be learned. Many of the undesirable symptoms can be eliminated with proper training. "Older persons present better possibilities for readjustment than a young person, because for half a century or more the older person has adjusted satisfactorily to the world. (6). What are possibilities of such training and which methods are the best to use? It is clear that there is still much to be learned about prefrontal lobotomy. We will conclude this paper by quoting Cobb's conservative opinion. "If the operation saves the patient from death or incurable psychosis, it is obviously worth while. If the disease is curable by other means or liable to remission, and if the patient is young, the operation seems to me unjustifiable." (2).

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CHEMISTRY

DERIVATIVES OF PENICILLIN AND OTHER MOLD PRODUCED ANTI-BACTERIAL SUBSTANCES

By JOSEPH A. DUKE, S.J.

The discovery of Penicillin and the phenomenal success it has had against injurious bacteria has encouraged research workers to further examine this field of antibacterial substances in search of others which might have advantages equal, or even greater than to Penicillin itself. The difficulties present in the production and preservation of Penicillin also give impetus to these investigations. A multitude of experiments simultaneously conducted in a host of laboratories must result in new discoveries. We shall consider some of these in our paper.

Our treatment shall group these new substances under a triple heading. First we shall discuss the chemical derivatives of Penicillin itself. Then we shall treat the substances other than Penicillin which have been extracted from the same mold from which Penicillin is obtained, i.e. *Penicillium notatum*. Finally, we shall mention the derivatives from other species of molds in the light of their antibacterial properties.

It was while searching for a form of Penicillin which would be more stable, and one which would not be so rapidly excreted by the patient that the esters of Penicillin were prepared and studied¹. Since the carboxyl groups were judged to be the cause of the liability of Penicillin it was argued that esterification would render the substance more stable. At first some unsuccessful attempts were reported². But subsequently three workers in New York were able to make the methyl, ethyl, n-butyl and benzohydril esters by reacting the free Penicillin acid with the corresponding diazo compound. These esters were insoluble in neutral or lightly alkaline buffers, showed a marked solubility in benzine and were not precipitated from chloroform-benzine solutions by dry ammonia, in which properties they are in sharp contrast to their mother acid.

In studying the activity of these esters it was discovered that, *in vitro*, they were not as active as it was hoped they would be. This fact was probably due to the hydrolysis of the esters by the bacteria. When injected into mice, however, these same esters showed a greater activity than when examined *in vitro* and justified the conjecture which had prompted their investigation.

A second substance extracted from the preparations of Penicillin was discovered through the increased optical activity of aqueous

solutions of Penicillin which were kept at pH 2 for some time. The solutions showed a marked d-rotatory property which reached a maximum. The ether extracts after this change had occurred were not as large as they were previously and there remained in the colorless water layer a strongly d-rotatory substance which fluoresced in ultra violet light. The substance was called Penillic acid and was isolated by means of butyl alcohol. The yields were in direct proportion to the activity of the original preparations. This last indicated that Penillic acid is a derivative of the active Penicillin itself and not of any concomitant substance.

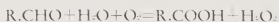
Penillic acid is obtained as brilliant rhombic crystals which decompose without charring but with gas evolution at 175°. In solution it exhibits a pale fluorescence in ultra violet light and shows a high positive specific rotation. Acid to litmus, the Penillic acid is precipitated by mercuric chloride and phospho-tungstic acid. It forms a silver salt which is slightly soluble. With ninhydrine a deep blue color results while it does not give the blue color characteristic of Penicillamine with ferric chloride.

The third extract from Penicillin preparations has been discovered by several investigators and has been given at least four names. Although some doubt remains it seems quite evident that there is but one substance in question. It has been called successively Penicillin A¹, Notatin², Penicillin B³, and Penatin⁴,⁵,⁶.

The British investigators gave the name Notatin to the substance which they obtained by concentrating the crude filtrate, precipitating the Notatin by acetone, and producing a complex with tannic acid from which the pure substance may be obtained by removing the tannic acid with acetone. Further purification is possible by decomposition of the complexes it forms with Reinecke salts. It is a water soluble, buff-colored powder which in aqueous solution has a pH of 6-7. Its activity is destroyed by pH value less than 2 and more than 8. Seventy percent aqueous methyl alcohol at 30° destroys Notatin as does trichloroacetic acid. The products of this fission are a proteinate precipitate and a solution of the prosthetic group neither of which has any antibacterial properties.

Notatin gives many protein tests and its absorption spectrum is that characteristic of a yellow enzyme. The evidence of its being a dinucleotide is given by the fact that its silver salt of the prosthetic group is insoluble.

Three conditions are required for Notatin to exert its powerful antibacterial properties. There must be oxygen and glucose present while appreciable amounts of catalase must be absent. The glucose is converted to gluconic acid with the production of hydrogen peroxide. The equation may be expressed:



Since the production of the hydrogen peroxide is decreased when catalase is present, Notatin exhibits properties very much like the glucose oxidase extracted from *Aspergillus niger* micelia.

At the end of their article the British scientists note the similarity between Notatin and Penatin (discovered at the University of Pennsylvania) but cite the insufficient evidence as the reason for no definitive statement regarding their identity.

In reporting his discovery of Penatin⁷ Kocholaty states that the mold which produces this substance in greatest quantities is not the *Penicillium notatum* Westling (that used by Fleming to produce Penicillin) but another strain of the same *Penicillium notatum* species. The technical name of this last is PEN 2. Penatin is said to possess far greater bacteriostatic properties even being effective against some gram-negative organisms, a class left unaffected by Penicillin. This last is however more bactericidal respecting the injurious organisms than is Penatin.

Kocholaty extracted Penatin from the crude *Penicillium* culture by precipitating with phosphotungstic acid forming the Penatin-phosphotungstate which is insoluble in acids. The purification of the latter compound could not be effected without a serious loss of the Penatin so a different process was devised for its purification. Penatin is absorbed on kaolin at pH 4 and is then removed by a solution of pyridin or sodium phosphate at pH 6.3. The Penatin is then precipitated by dioxane. It is a stable yellow powder, insoluble in water. In its action towards bases Penatin has been found to be very sensitive but less affected by acids.

In the London school of Hygiene and Tropical Medicine, Professor H. Raistrick and his colleagues¹⁰, discovered a substance which they called Patulin. This was analogous to Penicillin but came from a different mold of the same family, *Penicillium patulum*. Chemically this substance has been identified as anhydro-3-hydroxy-methylene-tetrahydro- γ -pyrone-2-carboxylic acid. It is a colorless plate-like crystal isolated from two strains of *Penicillium patulum*, one from Holland and the other found in condenser water. It is stable in a phosphate buffer at pH 6. Against bacteria it is less active than Penicillin for gram-positive but much more active toward gram-negative bacteria. The one malady against which Patulin has been used quite successfully is the common cold. Objective norms for estimating the effectiveness of this new substance in this regard, unfortunately, are lacking.

Experiments with the mold *Aspergillus fumigatus* var. *Helvola* have yielded helvolic acid, another anti-bacterial product¹¹. It is especially active against gram-positive bacteria and is more bacteriostatic than bactericidal. It is a crystalline substance insoluble in water but soluble in most organic solvents. Its sodium salt is, however, extremely water soluble.

Some experimenters at Rutgers University have discovered approximately 160 anti-bacterial organisms from different fungi¹². Two of this number were more extensively studied, *Aspergillus fumigatus* and *Aspergillus clavatus*. The anti-bacterial substances obtained from them were named fumigacin and clavacin respectively.

Fumagacin is very soluble in chloroform and ethyl alcohol and is removed from the crude culture medium by adsorbing it on norite, which has been previously treated with ether, and then removing the antibiotic by elution with chloroform. Distillation removes the eluting liquid and the fumagacin is crystallized from alcohol in long slender needles. It is soluble to some extent in ether and water. The authors note the distinction of fumagacin from fumagatin, a pigment isolated by Raistrick and his co-workers^{13, 14}.

Clavacin is prepared by adsorption of the substance on norite and removing it by means of ether or chloroform. A simpler method is to dissolve the clavacin directly in either of these solvents and extract it from the crude culture medium in this way. It is readily soluble in dilute alkalis and is active against gram-negative bacteria and is even bactericidal to a marked degree against gram-positive and gram-negative organisms.

A previous report¹⁵ of an antibiotic (probably identical with clavacin¹²) was made by Dr. Wiesner of the Royal Northern Hospital in London. He emphasizes the advantages of this substance over Penicillin in that the former is more stable, its filtrates are less easily contaminated and in high acidities the substance is not inactivated. Moreover it is not bacteriostatic but is also bactericidal and that for a greater variety of bacteria than is Penicillin.

This brief survey of the discoveries which have been made during research in antibiotics shows that this field promises to be the source of most of our post-war remedies for the numerous diseases caused by bacteria. The phenomenal successes obtained in the experiments have resulted in focusing the attention of thousands of scientists on a very ordinary type of organism, the molds. Where these advances shall stop no one can now predict, but we are certain that unless something quite unforeseen occurs, we have already conquered many of the ills to which preceding generations have been highly susceptible.

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A recent issue of the *Journal of Chemical Education* (21 503, 1944) carries as an illustration of an article entitled, "Salt, the Most Useful of Mineral Substances," a detail from Da Vinci's "Last Supper." It is interesting, and perhaps not common knowledge, that there is a salt cellar overturned by the side of Judas, as an omen of what is coming. This picture recalls to mind a picture in the same *Journal* some years ago (11 2, 1934) depicting Christ as the Apothecary. The picture is filled with symbolism, and the issue carries an article with full explanation and other illustrations.

MATHEMATICS

THE GRAPH OF THE CUBIC EQUATION

By REV. THOMAS D. BARRY, S.J.

In the course of my teaching I have had several occasions to draw the graphs of equations of the form $y = ax^3 + bx^2 + cx + d$. They always resulted in one of the shapes illustrated here. But whenever I wanted a graph with special characteristics, I automatically picked the wrong equation. So I decided to find out the connection between the coefficients of the equation and the type of the graph, with the results given here. It should be understood that the main purpose here is to give only a general idea of the shape of the graph, although some corollaries at the end will apply to other characteristics.

In the first place, $y = ax^3$ is the equation of a standard curve, the cubical parabola, in which the tangent at the point of inflection is horizontal. Its graph is intermediate between those of Types I and II. The addition of x -terms of lower degree will modify that curve as shown below. The only effect of the constant term is to raise or lower the curve $y = ax^3 + bx^2 + cx$, so it will not enter into the present discussion. Since the Type II curves have maxima and minima, whereas the Type I curves do not, the simplest way of attacking the problem is to apply the principles of curve tracing (cf. Granville's Calculus, p. 81). Briefly, the first derivative of the function is equated to zero and solved for real values of x . These values of x (called critical values) give the abscissas of the points where the curve may have maxima or minima. They are tested by substitution in the second derivative. If this becomes plus the function has a minimum, if minus, a maximum.

$$y = ax^3 + bx^2 + cx + d$$

$$\text{First derivative: } y' = 3ax^2 + 2bx + c = 0$$

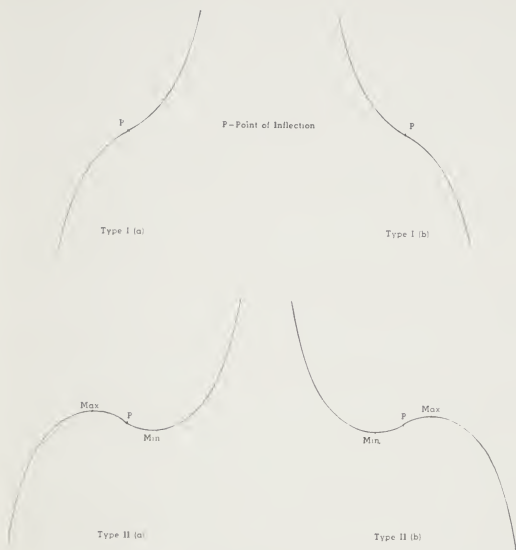
$$\text{whence } x = -\frac{b}{3a} \pm \sqrt{\frac{b^2}{9a^2} - \frac{3ac}{3a^2}}, \text{ the critical values. (1)}$$

$$\text{Second derivative: } y'' = 6ax + 2b \quad (2)$$

Substituting (1) in (2) we have

$$y'' = \pm 2 \sqrt{\frac{b^2}{9a^2} - \frac{3ac}{3a^2}}$$

Therefore, when (1) is real, there will be a minimum of the function for the plus sign, and a maximum for the minus sign. Parenthetically it may be noted that from the second derivative the point of inflection is found to be at $x = -\frac{b}{3a}$. By comparison with (1), it will be seen that the critical points are equidistant, right and left, from the point of inflection. Referring to (1):



Case I. If $b^2 > 3ac$, the radical, and hence the critical values, are real. Hence the function will have a minimum for the plus sign and a maximum for the minus sign. Therefore the graph will be one of the Types II. Which of these it is will be clear from the following. If a is plus, the minimum will be to the right of the point of inflection ($x = -b/3a$), and the maximum will be to the left. Therefore Type II (a). If a is minus, the positions will be reversed. Therefore Type II (b). The graph therefore will be of $y = ax^3$, modified, as it were, by pulling part of the right hand side of the curve (for plus a) below the point of inflection, and part of the other side above it. Conversely for minus a .

Case II. If $b^2 = 3ac$, the radical is zero, and the critical points coalesce in the point of inflection. It can be shown that in this case the equation can be reduced to the form $y - k = a(x - h)^3$, which is simply $y = ax^3$ moved to a new location. This is here classified as Type I, since there is no maximum or minimum.

Case III. If $b^2 < 3ac$, the radical, and hence the critical values are imaginary, hence no maximum or minimum. The graph will be Type I (a) or (b), according as a is plus or minus. In this case (for plus a , conversely for minus a) the right hand side of the graph may be considered to be pulled higher and the left hand side lower than in the case of $y = ax^3$.

The above results may be summed up as follows:

- 1) Considering the curve as the locus of a point moving from left to right, it will be a generally ascending curve, i. e., Type (a), if a is plus, and a generally descending curve, i. e., Type (b), if a is minus.
- 2) It will be Type II if $b^2 > 3ac$. This condition will be fulfilled:
 - a) in the equation $y = ax^3 + bx^2 + cx + d$, in general, if $b^2 - 3ac$ is a positive number, in particular, if a and c have opposite signs;
 - b) in the equation $y = ax^3 + bx^2 + d$, always, since $c = 0$;
 - c) in the equation $y = ax^3 + cx + d$, if a and c have opposite signs.
- 3) In all other cases the graph will be Type I.

From the above discussion, the following corollaries may be deduced.

I. Maxima and minima. If the coefficients show that the graph is Type I, it follows at once, without going through the procedure of the calculus books, that there can be no maximum or minimum of the function. If Type II, there must be.

II. Roots of the equation ($y =$) $ax^3 + bx^2 + cx + d = 0$. If the graph is Type I, there can be only one real root of the equation. If Type II, there may be one or three. (It can be shown that there will be three real roots if d lies between the values $(3abc - 2b^D \pm 2^D \sqrt{D}) / 27a^2$, where $D = b^2 - 3ac$. If d equals either end of that interval, two of the roots will be equal. This test may be applied fairly readily if the coefficients of the equation are not very large.)

III. Position of the curve. Since the point of inflection has the abscissa $x = -b/3a$, it will lie 1) to the right of, 2) to the left of, or 3) on the y -axis, according as 1) a and b have opposite signs or 2) the same sign, or 3) $b = 0$ (It can also be shown that the point of inflection will lie 1) above, 2) below, or 3) on the x -axis, according as d 1) is greater than, 2) is less than, or 3) equals $(3abc - 2b^D) / 27a^2$.)

Some examples, showing the application of the rules developed in the main part of this paper:

- 1) $y = x^3 + 3x^2 + 9x - 5$. a is plus, $b^2 < 3ac$. Type I (a).
- 2) $y = x^3 - 3x^2 + 5$. a is plus. Type II (a).
- 3) $y = 2 + 3x - 4x^2 - x^3$. a is minus, c is plus. Type II (b).
- 4) $y = 12x - x^3$. a is minus, c is plus. Type II (b).

PHYSICS

GENERALIZED OUTLINE FOR SPHERICAL MIRRORS

STANLEY J. BEZUSKA, S.J.

The following treatment is intended as a synopsis embracing the main statements about spherical mirrors in relation to the solution of problems. One fundamental equation is used for both concave and convex mirrors and is based on the assumption that the mirrors are sections of spherical surfaces with large radii of curvature. The analytic proof of the equation can be found in former articles of the BULLETIN which were developed by Rev. Thomas H. Quigley, S.J.¹

To facilitate the visualization of the symbols used in the discussion, a set of diagrams for concave and convex mirrors have been drawn. Cf. Figures 1, 2, 3, 4.

In these figures, OF=focal length of the mirror

OR=radius of curvature of the mirror=2(OF)

OD=distance of object AB from mirror

OP=distance of image A'B' from mirror.

The arrow lettered AB is always on the side of the reflecting surface, while its image is primed (A'B').

Thus, we have three figures for the concave mirror illustrating the possibilities that may occur, namely,

- a) the object AB may be beyond the radius of curvature
- b) between the focal length and radius of curvature
- c) in front of the focal point.

For the convex mirror, one diagram is sufficient, for no marked distinctions such as those for concave mirrors occur.

We take as our fundamental mirror equation:

$$\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{F} \quad (1)$$

whose variations for the solution of different terms are:

$$F = \frac{D_o \cdot D_i}{D_o + D_i} \quad (2)$$

¹ The Lens Formula. Bulletin A. A. J. S., Vol. VII, No. 4. Reflection and refraction of spherical surfaces. Bulletin A. A. J. S., Vol. X, No. 2.

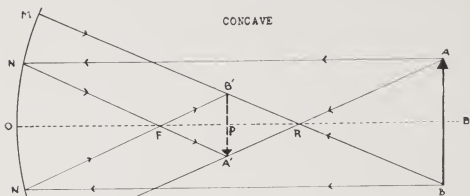


Fig. 1

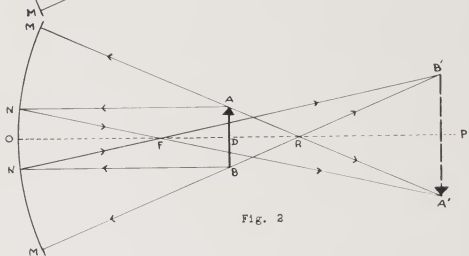


Fig. 2

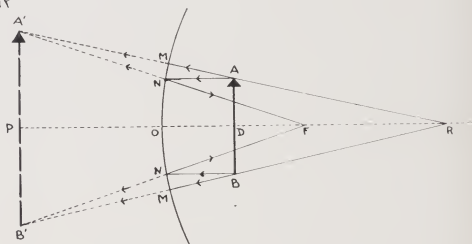


Fig. 3



Fig. 4

$$D = \frac{FD_i}{D_i - F} \quad (3)$$

$$D_i = \frac{FD_o}{D_o - F} \quad (4)$$

And to complete the discussion, we add:

$$S_i / S_o = D_i / D_o \quad (5)$$

$$M = D_i / D_o \quad (6)$$

Where F = focal length of mirror
 $2F$ = radius of curvature of mirror = R
 D_o = distance of object from mirror
 D_i = distance of image from mirror
 S_o = size of object
 S_i = size of image
 M = magnification factor.

CONCAVE MIRRORS:

In the case of concave mirrors, the image may be a combination of several of the following:

Real	Inverted
Virtual	Magnified
Erect	Reduced

depending on where the object is placed with regard to the focal length and radius of curvature of the mirror. Note however, that if the image is virtual, then D_i will come out negative from Eq. (4) if you are given F and D_o . On the other hand, if you are given the

Chart I. Concave Mirrors

D_o , F always positive.

D_o at Infinity, R , or F : image at: F , R , infinity (parallel rays) respectively

	Relations D_o , D_i , F , R	Image	Position of image.	D_i	
				If solving for D_i Eq. (4) comes out:	If subing in Eq. (2), (3) use:
1	$D_o > R > F$	Real Inverted Reduced	$F < D_i < R$	+	+
2	$R > D_o > F$	Real Inverted Magnified	$D_i > R$	+	+
3	$R > F > D_o$	Virtual Erect Magnified	$-D_i$ if: $D_o > F/2 < F$ then: $ -D_i > -F $ so also: $D_o < F/2 < F$ $ -D_i < -F $ $D_o = F/2$ $ -D_i = -F $	-	-
4	$D_i > D_o$	and $R > D_o > F$ Real Inverted Magnified	$D_i > R$	+	+
		and $F/2 < D_o < F$ Virtual Erect Magnified	$-D_i$ $ -D_i > -F $	-	-
5	$D_i < D_o$	and $D_o > R > F$ Real Inverted Reduced	$F < D_i < R$	+	+
		and $D_o < F/2$ Virtual Erect Magnified	$-D_i$ $ -D_i < -F $	-	-

fact that the image is virtual, and asked to find either F or D_o , then in Eqs. (2) and (3) a minus sign must be used before D_i . A negative D_i also means that the image is on the opposite side of the reflecting surface and therefore virtual. D_o and F will always be positive.

CONVEX MIRRORS:

For convex mirrors, the image will always be

Virtual

Erect

Reduced

Since the focal length in this case is taken on the opposite side of the reflecting surface, it is considered virtual and a negative sign must be used before F in Eqs. (3), (4), and F will come out negative in Eq. (2). D_i will come out negative if we use Eq. (4), and a negative sign must be used before D_i in Eqs. (2), (3).

Chart II. Convex Mirrors

Image	D_o (always positive)	D_i	F (virtual focus)
6 Virtual	For all D_o and Erect Reduced	Use negative sign before D_i in Eqs. (2), (3) D_i comes out neg. in Eq. (4)	Use negative sign before F in Eqs. (3), (4) $-F$
7 Erect	For all D_o and Virtual Reduced		
8 Reduced	For all D_o and Virtual Erect		

$$D_o = F, \text{ the image is at } D_i = -F/2$$

$$D_o = R, \text{ the image is at } D_i = -2F/3$$

$$D_o \text{ at infinity, image is at } D_i = -F$$

$$\text{No matter where the object is placed before the mirror: } |D_i| \leq |F|$$

Chart III.

Given: $F, M.$

$$M = a/b = D_i/D_o$$

$$D_o = bD_i/a, \quad D_i = aD_o/b$$

<u>Concave Mirror:</u>		<u>Real Image:</u>	<u>Find:</u>
9	D_o	D_i	
	Use: $+F$	Use: $+F$	
	$D_i = aD_o/b$ and sub. in Eq. (3).	$D_o = bD_i/a$ and sub. in Eq. (4)	

<u>Concave Mirror:</u>		<u>Virtual Image:</u>	<u>Find:</u>
10	D_o	D_i	
	Use: $+F$	Use: $+F$	
	$D_i = -aD_o/b$ and sub. in Eq. (3).	$D_o = -bD_i/a$ and sub. in Eq. (4).	

<u>Convex Mirror:</u>		<u>Virtual Image:</u>	<u>Find:</u>
11	D_o	D_i	
	Use: $-F$	Use: $-F$	
	$D_i = -aD_o/b$ and sub. in Eq. (3).	$D_o = -bD_i/a$ and sub. in Eq. (4).	

There are a few critical cases for the concave and convex mirrors, namely,

1. when the object is at infinity (i. e. at a great distance in comparison with the focal length)
2. when the object is at the radius of curvature
3. when the object is at the focal point.

These cases as well as the general summary are given in Charts I, II, and III.

To exemplify the use of the charts, two problems are given.

A) A concave spherical mirror has a focal length of 20 cms. How far and what kind of an image is formed if the object is

- a) 60 cms. away
- b) 25 cms. away
- c) 10 cms. away.

What is the magnification factor in each case?

SOLUTION:

- a) $F = 20$ cms.
 $R = 2F = 40$ cms.
 $D_o = 60$ cms.

Therefore, $D_o > R > F$

Chart I, No. 1, the image is Real, Inverted, Reduced
 it lies between F and R.

D_i should come out positive from eq. (4)

Calculation of eq. (4) gives: $D_i = 30$ cms.

Calculation of eq. (6) gives: $M = \frac{1}{2}$

- b) $D_o = 25$ cms.

Therefore, $R > D_o > F$

Chart I, No. 2, the image is Real, Inverted, Magnified
 it lies beyond R

D_i should come out positive from eq. (4)

Calculation of eq. (4) gives: $D_i = 100$ cms.

Calculation of eq. (6) gives: $M = 4$

- c) $D_o = 10$ cms.

Therefore, $R > F > D_o$

Chart I, No. 3, the image is Virtual, Erect and Mag
 nified it lies at $-F$ since $D_o = \frac{F}{2}$ cf. Chart I, No. 3

D_i should come out negative from eq. (4)

Calculation of eq. (4) gives: $D_i = -20$ cms.

Calculation of eq. (6) gives: $M = 2$

B). A convex spherical mirror has a focal length of 20 cms. How far from the mirror is the image when the object is

- a. 80 cms. away
- b. 30 cms. away
- c. 20 cms. away

From Chart II, the image for all three cases is virtual, erect and reduced. Since we are looking for D_i , it will come out negative in eq. (4), and F is also negative.

a) $F = -20$

$D_o = 80$ and from eq. (4), $D_i = -16$ cms.

b) $F = -20$

$D_o = 30$ and from eq. (4), $D_i = -12$ cms.

c) $F = -20$

$D_o = 20$ and from eq. (4), $D_i = -10$ cms.

